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Solving the optimal location problem in forest fire control with fuzzy data points

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Abstract

In this paper, we present a methodology to solve location problems when the data used is inherently fuzzy. This method, from data clusterized with the fuzzy $c-$means algorithm, calculates bi-dimensional fuzzy numbers from the clusters which are used to calculate a fuzzy solution. We apply the methodology, with different objective functions, to a particularly apt data set of forest fire breakouts in the Bouches du Rhone region of southern France, gathered from 1981 to 2009. The robustness of the method is then evaluated with a Monte Carlo simulation in which the number of clusters change. The solution provided with this fuzzy method provides leeway to planners, which can see how the membership function of the fuzzy solution can be used as a measurement of "appropriateness" of the final location.
Keywords: Forest fires, location problem, fuzzy sets, robustness.

1 Introduction

The application of GIS in studying the forest fire problem was implemented from different perspectives ranging from hill fire impact [4], mapping fire regime across time and scale [8] and spatial data for national fire planning and fuel management [5]. In particular, the control of forest fires has been a subject of research in which the application of GIS has been recurrently used [3][10].

Nevertheless, traditional operations research methods implemented in commercially available GIS software to solve the location problem, like the $p-$median and p-center, have considered that the data provided to optimization methods are always crisp and accurate. Then, the solution obtained in the process is accurate and crisp, or at least a finite set of predefined locations. Depending on the objective of the planner, the three most frequently used solutions, that have a closed form expression, are the barycenter, which takes into consideration equity objectives, the median center, that looks for robustness of the solution to changes in the data points, and the min-max center, that minimizes the maximum distance.

But there are some cases in which the data is fuzzy, so it is unreasonable to obtain a crisp solution as it might be far from the optimum. Because the data points are fuzzy, the distances between them will also be fuzzy, which in turn casts uncertainty over the final solution. In the case of geography this is particularly true when the location of the data points is obtained through subjective or vague information. In this paper, we adapt a traditional line of research with the inclusion of fuzziness given by the fuzzy nature of the data points. Thus, following this line of research, we look for the fuzzy center that minimizes the sum of fuzzy distances to the fuzzy points.

Beyond a non negligible interest about risk management in forest fire breakouts, these two methodological statements, the objective pursued and the fuzziness of the data, are the main reasons why we decided to undertake this work in particular. Indeed, we make the assumption that, including fuzziness in the distance and hence in the final center location, we can provide a different perception of an optimal location to planners and decision makers, one that would include data uncertainty.

In this paper, fuzzy information about the location of forest fire breakouts in the Bouches du Rhone department is used to solve the fuzzy location problem of resources to control them. This dataset has been customarily used to build empirical knowledge throughout the years. The objective of this paper has been to present those responsible of forest fire control in the region of study, our operational partners in this project, with a methodology adapted to the data they have. We want to provide them, in first place, the possibility of comparing the results obtained using the new methodology with those achieved by their customary process, and secondly, leeway in their decision making process by using a solution that considers an area and not just one point.

To do so, we propose to search an optimal center location within a continuous space (without considering road networks), using historic data on forest fire breakouts. This center could be used for surveillance and management, including as a helicopter base (and possibly small planes) for quick intervention on fires in non-urbanized areas. From the methodological perspective, we also want to evaluate how the method behaves when we induce changes in the way the data set is used, evaluating its robustness in the process.

This paper is structured in the following way. In Section 2 we will describe the area of study, and the data that will be used in our calculations. The methodology, including some elementary framework of the fuzzy sets theory as well as the fuzzy centers that are calculated, is presented in Section 3. Finally, a discussion on the methodology based on simulation results and some conclusions are drawn in Section 4.

2 Area of study

In southern France, the Bouches du Rhone region is very sensitive to the forest fire risk. Its Mediterranean climate is characterized by a long hot and dry summer period with frequent and sometimes violent winds. Limestone grounded vegetation offers a weak water retention capacity, particularly in small mountainous hills. This vegetation mainly consists of resinous trees (*Pinus halepensis*) and shrub (*Quercus coccifera*, *Cistaceae*, *Rosmarinus officinalis*, *Thymus vulgaris*), in which the shrubby stratum changes from quasi steppe (herbaceous) to bush type vegetation (*Juniperus communis*, *Quercus* residuals). Most of these species have low-water content and can be highly inflammable when a period of hydric stress occurs. Depending on the vertical and horizontal configuration of the vegetation, forest fires can turn out to be powerful, hard to fight against and can damage several thousands of hectares.

What makes this region different is that the urban and vegetation areas are strongly inter-weaved in a highly populated region, with a busy and developed transport network. As the forest fire risk is highly correlated with the proximity to roads, this area is prone to fires[1] close to residential areas, which increase the difficulty of firefighting. Nonetheless, there are some areas dedicated to agricultural activity that can first play a role in preventing fires, but that can also participate in the spreading of fire due to wildland patches linked to the decline of the activity. This configuration is quite original when compared to other regions of the world suffering from a high fire risk, but with bigger and homogeneous forests.

Therefore, the heterogeneous and rather complex layout of this risk-sensitive environment offers a good experimental ground in which to test hypothesis about the optimal location of intervention centers, as well as an operational pre-positioning of both material and staff in high-risk time. Indeed, this territory has been organizing the protection against forest fires for a long time, hosting the *Défense de la Forêt contre les Incendies'* (DFCI) *Centre Opérationnel de Zone* (COZ), the *Entente pour la Forêt Méditerranéenne* in Gardanne, and an aerial base in Marignane. Aerial fighting resources are frequently the first to intervene, thanks to a monitoring system called "armed aerial watch". Then, the ground team must end the quenching of the fire after aerial water dropping.

2.1 Data

The Bouches du Rhone region is part of the *Prométhée* project that, since 1973, has been updating a database, managed by the *Délégation à la Protection de la Forêt Méditerranéenne* (DPFM). This database holds the inventory of forest fire breakouts that occurred in the French Mediterranean area (80 000 km²). The data is represented in the DFCI (*Défense Forestière Contre les Incendies*) system [6], a space partition which defines square areas of 4km^2. Each forest fire breakout is located inside the square in which it occurred, along the date, burnt surface and some complementary information (proximity to an anthropogenic structure, vegetation type, total surface of threatened vegetation and the cause).

As we can see, the data is spatially fuzzy, as only an approximate location of the breakout is available. Inside each square of the grid, only the number of fires is available. We can use a traditional method in which each square is treated as an individual data point, weighted by a combination of the number of fires and the area burnt. The solution following these traditional methods will be a point, something that will leave the decision maker with no flexibility as moving away from the solution is sub-optimal. Our objective is to use the fuzzy nature of the data, providing a fuzzy solution that defines an area, not just a point. Any point in this area gives the planner a value of "appropriateness" of the final location of the center, telling him how much will he loose by deviating from the most valued area.

From a semantic point of view, and for the purpose of this paper, we should specify that a forest fire is defined as a fire concerning or directly threatening a forest area of at least 1 hectare. Thus, the *Prométhée* database considers forest fires in the strict sense of the term. Though this definition is justified by the land use, it is a French characteristic which does not seem perfectly compatible with international standards but offers the opportunity to enable a deep study of the risk. Therefore, we made the choice to keep only the events that really impacted a forest in the Bouches du Rhone region between 1981 and 2009, i.e., 337 statistical observations. In order to apply our methodology, the coordinates of the southwestern corner of each square of the DFCI system than covers the area of study was reprojected into the Lambert II Extended system, following the algorithm presented in [6, pags. 7–8] and which was coded in GNU R.

3 Methodology

3.1 Clustering

As we will see, to provide a fuzzy solution the methodology relies on the definition of fuzzy data points, areas in which a membership function can be expressed. Among the many clustering algorithms that are available, we selected the fuzzy c-means algorithm [1], which requires as initial parameter the number of clusters to be build. In discussions with the experts, and it was decided, in a first step, that according to the area of study a set of 20 different clusters of rectangular shape was adequate. The variables selected for the clustering process were the coordinates of the southwestern corner of each area and the number of fires that started in it. The results were reevaluated by the experts in order to eliminate partial overlapping of the clusters, reassigning some areas from the cluster they were originally assigned to neighboring ones. The final distribution of the clusters can be seen in Figure 2, with a statistical summary presented in Table 1. It is clear that though clusters vary in a considerable way in

[1] Even if they often are small, fires in the region can potentially become of a sizable importance [7].

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Table 1: Statistical summary of the clusters.

Cluster	Fires	Area (Km^2)	Density ($Fires/Km^2$)
1	14	144	0.10
2	16	324	0.05
3	3	16	0.19
4	38	144	0.26
5	7	144	0.05
6	5	36	0.14
7	23	144	0.16
8	23	100	0.23
9	17	100	0.17
10	19	196	0.10
11	18	196	0.09
12	5	64	0.08
13	14	64	0.22
14	23	64	0.36
15	6	36	0.17
16	21	196	0.07
17	17	256	0.08
18	15	196	0.17
19	25	144	0.19
20	28	144	0.19
Total	337	2748	0.12

size and, hence, in number of fires, the density of fires per Km^2 is smoothly distributed in its range, with the exception of cluster 14, which is 38% denser than the second densest (cluster 4).

3.2 Fuzzy numbers

The fuzzy sets theory was developed by Zadeh [11] as a way to work with uncertainty in a non-probabilistic environment. A fuzzy subset $\underset{\sim}{A}$ can be represented by a set of pairs composed of the elements x of the universal set X, and a grade of membership $\mu_{\underset{\sim}{A}}(x)$:

$$\underset{\sim}{A} = \left\{ \left(x, \mu_{\underset{\sim}{A}}(x)\right) \mid x \in X \,,\, \mu_{\underset{\sim}{A}}(x) \in [0,1] \right\}. \tag{1}$$

For the purpose of this paper, a fuzzy number is defined as a normal, convex fuzzy subset with domain in $\mathbb{R}$ for which $\mu_{\underset{\sim}{A}}$, its membership function, is at least piecewise continuous. The membership function of a trapezoidal fuzzy number (TrFN) is defined as:

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 1 - \frac{x_2 - x}{x_2 - x_1}, & \text{if } x_1 \leq x < x_2 \\ 1, & \text{if } x_2 \leq x \leq x_3 \\ 1 - \frac{x - x_3}{x_4 - x_3}, & \text{if } x_3 < x \leq x_4 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

This kind of fuzzy interval represents the case when the maximum of presumption, the modal value, can not be identified in a single point, but only in an interval between x_2 and x_3, decreasing linearly to zero at the worst case deviations x_1 and x_4. The TrFN is represented by a 4-tuple whose first and fourth elements correspond to the extremes from where the membership function begins to grow, and whose second and third components are the limits of the interval where the maximum certainty lies, i.e., $\underset{\sim}{A} = (x_1, x_2, x_3, x_4)$. For simplicity reasons, from now on we will use the lowercase version of the TrFN to identify its components, i.e., $\underset{\sim}{A} = (a_1, a_2, a_3, a_4)$.

The arithmetic of fuzzy numbers is derived from interval arithmetic. The addition, which is defined as:

$$\underset{\sim}{A} \oplus \underset{\sim}{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4), \tag{3}$$

while the product by a constant is defined as:

$$\alpha \cdot \underset{\sim}{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4).$$

Comparing fuzzy numbers is a task that can be simple to achieve through some method of defuzzification. For its simplicity and properties, we have selected the graded mean integrated representation (GMIR) of a TrFN [2] as the method used in this paper to defuzzify and compare TrFN. The GMIR of a TrFN is:

$$E\left(\underset{\sim}{A}\right) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}. \tag{4}$$

From this definition, we can the build a k-th order statistic. For a set $P = \{\underset{\sim}{p^{(i)}}\}$, $\forall i = 1, \ldots, n$, of TrFN, the $k-$th order statistic $\underset{\sim}{p^{([k])}}$ is defined as the $k-$th point for which $E(\underset{\sim}{p^{([k])}}) \leq E(\underset{\sim}{p^{([k+1])}})$. We now use the k-th order statistic to define the median of a set $P = \{\underset{\sim}{p^{(i)}}\}$, $\forall i = 1, \ldots, n$, of TrFN:

$$\text{median}(P) = \begin{cases} \frac{\omega_{[\frac{n}{2}]} \cdot \underset{\sim}{p^{([\frac{n}{2}])}} \oplus \omega_{[\frac{n}{2}+1]} \cdot \underset{\sim}{p^{([\frac{n}{2}+1])}}}{\omega_{[\frac{n}{2}]} + \omega_{[\frac{n}{2}+1]}}, & \text{if } n \text{ is odd,} \\ \underset{\sim}{p^{\left([\frac{n+1}{2}]\right)}}, & \text{if } n \text{ is even.} \end{cases} \tag{5}$$

where usually the weights are equal, i.e., $\omega_{[\frac{n}{2}]} = \omega_{[\frac{n}{2}+1]} = 1$.

3.3 Fuzzy distance

The fuzzy Minkowski family of distances between two fuzzy n-dimensional vectors $\underset{\sim}{A} = \left(\underset{\sim}{A_1}, \underset{\sim}{A_2}, \ldots \underset{\sim}{A_n}\right)$ and $\underset{\sim}{B} = \left(\underset{\sim}{B_1}, \underset{\sim}{B_2}, \ldots \underset{\sim}{B_n}\right)$ composed of TrFN:

$$\underset{\sim}{d_p}\left(\underset{\sim}{A}, \underset{\sim}{B}\right) = \left(\sum_{i=1}^{n} \left(\left| \underset{\sim}{A_i} \ominus \underset{\sim}{B_i} \right| \right)^p \right)^{\frac{1}{p}}. \tag{6}$$

As with the crisp Minkowski family of distances, the fuzzy Manhattan distance is defined for $p = 1$ and the fuzzy Chebyshev distance is defined for $p = \infty$.

$$\underset{\sim}{d_1}\left(\underset{\sim}{A}, \underset{\sim}{B}\right) = \sum_{i=1}^{n} \left(\left| \underset{\sim}{A_i} \ominus \underset{\sim}{B_i} \right| \right), \tag{7}$$

$$\underset{\sim}{d_\infty}\left(\underset{\sim}{A}, \underset{\sim}{B}\right) = \underset{\left|\underset{\sim}{A_i} \ominus \underset{\sim}{B_i}\right|}{\arg} \max_{i=1}^{n} E\left(\left| \underset{\sim}{A_i} \ominus \underset{\sim}{B_i} \right| \right). \tag{8}$$

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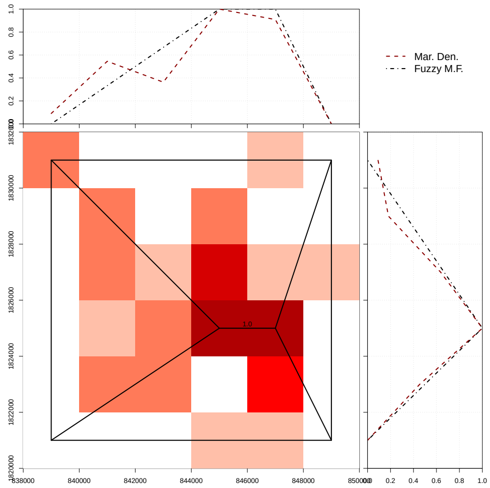
Figure 1: Example of a cluster's fuzzification in x and y. Red dashed line is the marginal density (Mar. Den.) and dash-dotted line is the fuzzy membership function (Fuzzy M.F.).



Algorithm 1 TrFN from normalized marginal average density.

1. Set the threshold value $\alpha \in [0,1]$.
2. $x_{c,2} = \min\left(x_c : \phi_x(c,x_c) \geq \alpha\right)$.
3. $x_{c,3} = \max\left(x_c : \phi_x(c,x_c) \geq \alpha\right)$.
4. $x_{c,1} = \begin{cases} \min(x_c{:}0<\phi_x(c,x_c)\leq\alpha, x_c\leq x_{c,2}), & \text{if } \nexists x_c{:}\phi_x(c,x_c)=0 \vee x_c\leq x_{c,2} \\ \max(x_c{:}\,\phi_x(c,x_c)=0, x_c\leq x_{c,2}), & \text{else.} \end{cases}$
5. $x_{c,4} = \begin{cases} \max(x_c{:}0<\phi_x(c,x_c)\leq\alpha, x_c\geq x_{c,3}), & \text{if } \nexists x_c{:}\phi_x(c,x_c)=0 \vee x_c\geq x_{c,3} \\ \min(x_c{:}\,\phi_x(c,x_c)=0, x_c\geq x_{c,3}), & \text{else.} \end{cases}$
6. $\utilde{x_c} = \left(x_{c,1}, x_{c,2}, x_{c,3}, x_{c,4}\right)$.

Table 2: Solutions.

Method	x_1	x_2	x_3	x_4
Median	831000	832243	834513	837000
Min-Max	836796	8379501	839106	845725
Barycenter	836796	837951	839106	845725

Method	y_1	y_2	y_3	y_4
Median	1833000	1833000	1841000	1846364
Min-Max	1853986	1853986	1853986	1857301
Barycenter	1833858	1836355	1837994	1843358

3.4 Fuzzyfication

The clustered data was the subject of a fuzzification process, an example of which can be seen in Figure 1. Given the boundaries $X_c = [\min(x_c), \max(x_c)]$ and $Y_c = [\min(y_c), \max(y_c)]$ of the cluster c, where x_c (conversely y_c) is a coordinate in the X (conversely Y) axis of the Lambert II Extended system, the number of 4 km^2 squares that compose each side of c is defined as:

$$n_{X_c} = \frac{(\max(x_c) - \min(x_c))}{2000} \tag{9}$$

$$n_{Y_c} = \frac{(\max(y_c) - \min(y_c))}{2000} \tag{10}$$

Given $f_{x,y}$, the number of fires in the DFCI 4 km^2 square whose southwestern corner is located in the coordinates $\{x,y\}$ of the Lambert II system, the marginal average density of the fires in c is defined as:

$$F_x(c,x_c) = \frac{1}{n_{Y_c}} \sum_{j=0}^{n_{Y_c}-1} f_{x_c, 2000\cdot j+\min(y_c)} \quad \forall x_c \in \{2000\cdot i+\min(x_c) : i=0,\ldots,(n_{X_c}-1)\} \tag{11}$$

$$F_y(c,y_c) = \frac{1}{n_{X_c}} \sum_{i=0}^{n_{X_c}-1} f_{2000\cdot i+\min(y_c), y_c} \quad \forall y_c \in \{2000\cdot j+\min(y_c) : j=0,\ldots,(n_{Y_c}-1)\} \tag{12}$$

The normalized version of the density is, thus:

$$\phi_x(c,x_c) = \frac{F_x(c,x_c)}{\max(F_x(c,x_c))}\,;\, \phi_y(c,y_c) = \frac{F_y(c,y_c)}{\max(F_y(c,y_c))}. \tag{13}$$

From the normalized version of the density for each coordinate axis, we obtain $\utilde{c} = \left\{\utilde{x_c}, \utilde{y_c}\right\}$, a fuzzy bidimensional number composed of TrFN calculated following Algorithm 1, where $\alpha \in [0,1]$ is a threshold that signals the range of the coordinate axis that belongs to the maximum of certainty of the proposition "the center of cluster c is located in $\{x_c, y_c\}$.

3.5 Fuzzy center

Once the data is fuzzified, we can proceed to solve the location problem according to the results presented in [9]. The first result states that for a set $C = \left\{\utilde{c}^{(i)} = \left\{\utilde{x_c}^{(i)}, \utilde{y_c}^{(i)}\right\}, \forall i = 1,\ldots,n\right\}$, where $\utilde{x_c}^{(i)}$ and $\utilde{y_c}^{(i)}$ are TrFN, and a set $\Omega = \{\omega_i\}$ of weights for the fuzzy points $\utilde{c}^{(i)} \in C$, the fuzzy median center is defined as:

$$\utilde{M} = \left\{\text{median}\left(\utilde{x_c}^{(i)}\right), \text{median}\left(\utilde{y_c}^{(i)}\right)\right\}. \tag{14}$$

The generalization of the second result shows that the fuzzy min-max center is defined as:

$$\utilde{Z} = \left\{\frac{\omega_{[1]}\cdot \utilde{x_c}^{([1])} \oplus \omega_{[n]}\cdot \utilde{x_c}^{([n])}}{\omega_{[1]}+\omega_{[n]}}, \frac{\omega_{[1]}\cdot \utilde{y_c}^{([1])} \oplus \omega_{[n]}\cdot \utilde{y_c}^{([n])}}{\omega_{[1]}+\omega_{[n]}}\right\}. \tag{15}$$

Likewise, it is possible to extend the result of the crisp weighted barycenter to the fuzzy environment:

$$\utilde{G} = \left\{\sum_{i=1}^{n} \frac{\omega_i \cdot \utilde{x_c}^{(i)}}{\sum_{i=1}^{n}\omega_i}, \sum_{i=1}^{n} \frac{\omega_i \cdot \utilde{y_c}^{(i)}}{\sum_{i=1}^{n}\omega_i}\right\}. \tag{16}$$

3.6 Solutions

The solutions for the clustered data, using a weighting set based on the density of fires per Km^2, are shown in Table 2, as well as in Figure 2. Due to the high concentration of fires in the center of the study area, both, the fuzzy median center and the fuzzy barycenter, are practically overlapped. Nonetheless, their maximum of presumption is completely different, as it is a long north-south strip for the fuzzy median center and just an small

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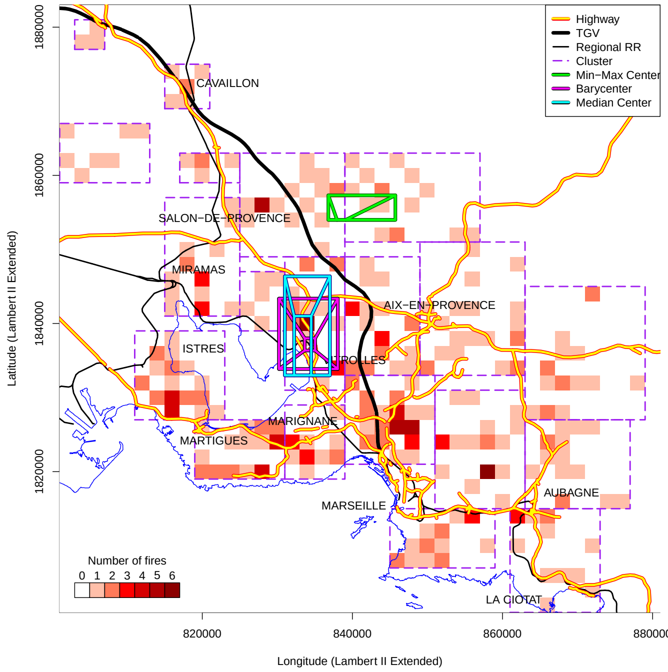
Figure 2: Data, clusters and solutions with weighting based on fire density per Km^2.



square for the fuzzy barycenter. Both of them are located in an area close to main roads and the Étang de Vaine, a lake that can be used for operations with helicopters and planes. On the other hand, the fuzzy min-max center is completely detached, in an area lacking both, high speed roads and water, north of the Luberon mountains. It seems that for this particular data set, minimizing the maximum risk gives a solution that can be problematic in terms of operations, raising the need to provide the necessary infrastructure to undertake them.

4 Discussion and conclusions

The main topic of discussion in the proposed methodology is the selection of the size of the clusters, which is directly related to the number of clusters fed to the clustering algorithm. It is directly inferable that smaller clusters produce solutions which are also smaller in area. Conversely, larger clusters produce solutions of larger area. Smaller (conversely, larger) solutions imply less (conversely, more) uncertainty, but also less (conversely, more) flexibility. Thus, it is of the uttermost importance to select clusters of the appropriate size, both by a clustering algorithm that follows very detailed constraints (like a minimum threshold for the density of fires per Km^2) and manual fine tuning by experts' knowledge. Nonetheless, we decided to carry out a Monte Carlo simulation (250 replications per case) in which the effect of the number of clusters (10, 20 and 40 clusters defined with the fuzzy $c-$means algorithm, without any *a posteriori* expert adjustment) is observed over the different methods of solution. The mean of the results, for each case and component, are presented in Table 3. The difference between the extreme components as well as between the inner components decreases when the number of clusters grows.

We can also see in this Table that the results for the cluster defined according to expert supervision are very close to the mean results of the simulations, except in one particular case, the y fuzzy coordinate of the fuzzy min-max center. The solutions in this case follow a bimodal density, with the biggest peak closer to the mean value presented in the table, and a smaller one closer to the solution obtained with the supervised clusters.

Another issue that arises from the methodology is about variability or robustness of the solutions, depending on the clusters defined for a particular data set. In the same simulation, we have computed the interquartile range (IQR) of the solutions (Table 4). The variability of the location of each component, in either coordinate axis, is fairly limited, except for the case of the fuzzy min-max center, which is usually more robust, but when changes appear they can be very drastic. This behavior is also inherent to the fuzzy median center, but as the data in our case is concentrated in the center clusters, the variability is reduced. Using this same Table, it is possible to infer the proper number of clusters to be used, what we can define as the optimal scale. For the case of both the fuzzy min-max and the fuzzy barycenter it is clear that by using 20 clusters we obtain the lowest variability. Unfortunately, in our dataset this is not so clear for the fuzzy median center.

We conclude that this methodology, if properly carried out, can give planners, like those in forest fire departments, liberty to select their particular solution from a set proposed as the result, obtaining its degree of "appropriateness" by the value of the membership function in the point selected. This methodology is an extension of traditional (crisp) methods used for location, which are derived from the minimization of an objective

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Table 4: IQR for each component.

Method (# Clusters)	x_1	x_2	x_3	x_4
Median (10)	4768	2069	8184	7445
Median (20)	2520	4459	4333	4005
Median (40)	4420	2924	2480	3000
Min-Max (10)	13068	20007	20776	20749
Min-Max (20)	8909	5184	8976	5750
Min-Max (40)	16606	16931	14920	12662
Barycenter (10)	6248	4138	6544	6514
Barycenter (20)	2391	2744	2904	2654
Barycenter (40)	5088	4872	4943	4787
Method (# Clusters)	y_1	y_2	y_3	y_4
Median (10)	1267	2152	2293	4462
Median (20)	2935	754	1379	2651
Median (40)	2536	596	805	5436
Min-Max (10)	9821	15361	8544	13679
Min-Max (20)	4021	1747	2394	5630
Min-Max (40)	34248	33371	31543	30667
Barycenter (10)	3958	4893	2878	3603
Barycenter (20)	2003	2466	2664	2345
Barycenter (40)	3441	3575	3532	3695

Table 3: Mean result for each component.

Method (# Clusters)	x_1	x_2	x_3	x_4
Median (10)	830217	836903	842524	850289
Median (20)	836061	840351	841901	845231
Median (40)	839530	841639	842549	844053
Min-Max (10)	829726	838432	839897	846324
Min-Max (20)	839399	840685	843797	846260
Min-Max (40)	844600	844782	846197	847195
Barycenter (10)	828157	834745	837857	844089
Barycenter (20)	832985	837011	838390	841595
Barycenter (40)	837820	839493	840151	841699
Method (# Clusters)	y_1	y_2	y_3	y_4
Median (10)	1824971	1829028	1831532	1844626
Median (20)	1829839	1832573	1832942	1839516
Median (40)	1831213	1832825	1833108	1835962
Min-Max (10)	1824549	1828635	1832038	1840514
Min-Max (20)	1824014	1825525	1826644	1832182
Min-Max (40)	1834740	1836445	1837303	1839773
Barycenter (10)	1825087	1829204	1832724	1841455
Barycenter (20)	1828158	1830826	1831769	1837114
Barycenter (40)	1830756	1832277	1833029	1835262

function that models the objective to achieve (equity, robustness, minimization of maximum risk), making it easier to understand and interpret.

Besides, this mathematical model offers the operational and geographical interest of allowing an adaptation of the multi-scaled analysis to the forest fire risk. It becomes possible to study and follow for several years, the movement of the center of gravity of the fires in the study area, in order to adapt the preventive and fighting actions to changes within the spatial pattern of the phenomenon.

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